MSc projects at BT Research

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Adastral Park, Martlesham, Suffolk

- Cambridge-Ipswich high-tech corridor
- 2000 technologists
- 15 companies
- UCL, Univ of Essex

BT Research centres

- Broadband Centre
- Foresight Centre
- IT futures research Centre
- Intelligent systems Centre
- Mobility Centre
- Networks Centre
- Pervasive ICT Centre
- Security Centre
- Asian Research Centre
- Disruptive lab at MIT

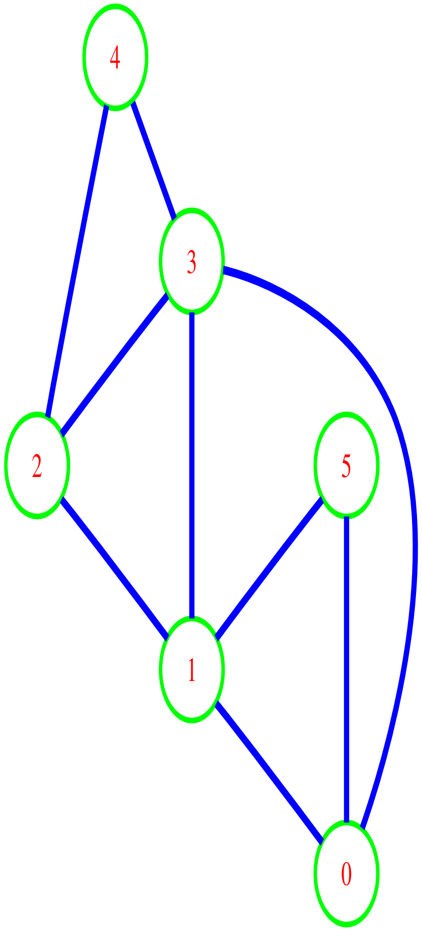
MSc projects supervised by Keith Briggs

Details: `www.keithbriggs.info/MSc_project_ideas_2008.html`

Mostly in discrete maths, two in ODEs. . .

- *1. Semidefinite programming and graph theory*
- *2. Singular value decomposition updating*
- *3. Singular value decomposition methods for Lyapunov exponents*
- *4. Automatic differentiation (AD) methods for ODE sensitivity analysis*
- *5. Fast random selection*
- *6. Random sampling of set partitions*
- *7. Fast counting*
- *8. Random sampling of unlabelled structures*
- *9. Convex optimization in python*
- *10. Geographical computations*
- *11. Statistics of Roman roads in Britain*

Graph concepts



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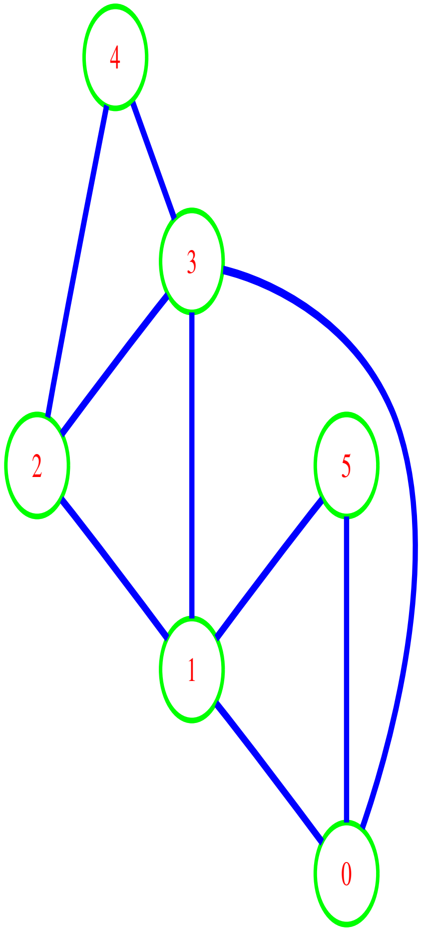

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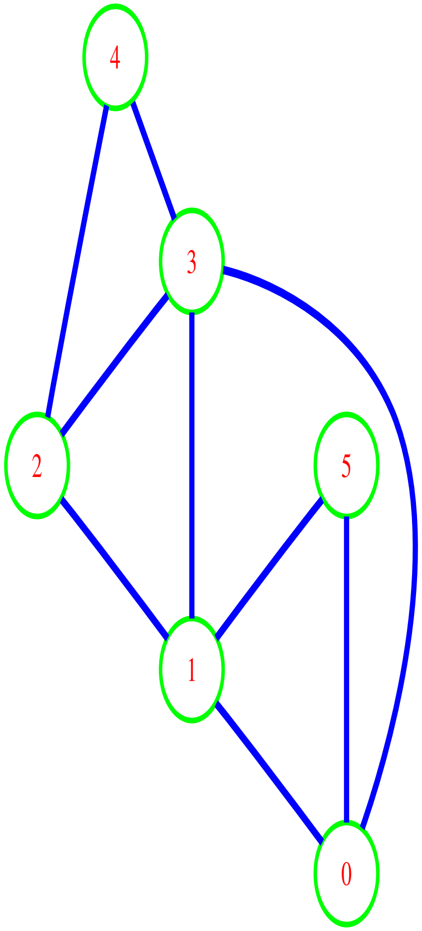

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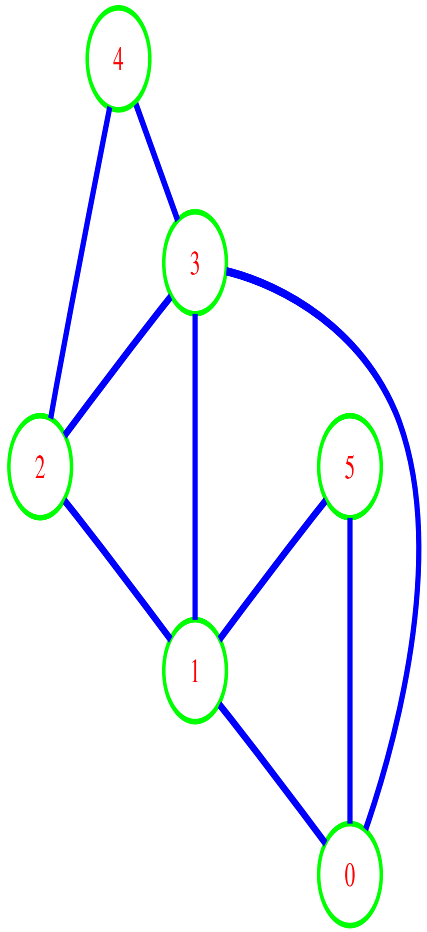

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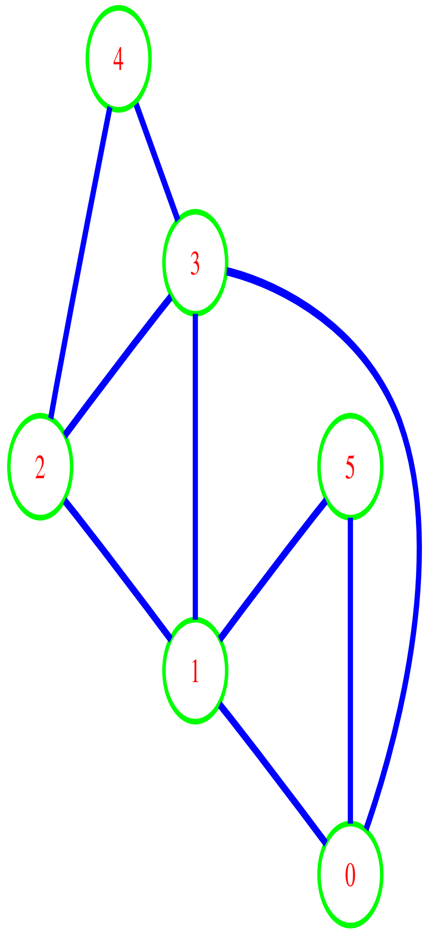

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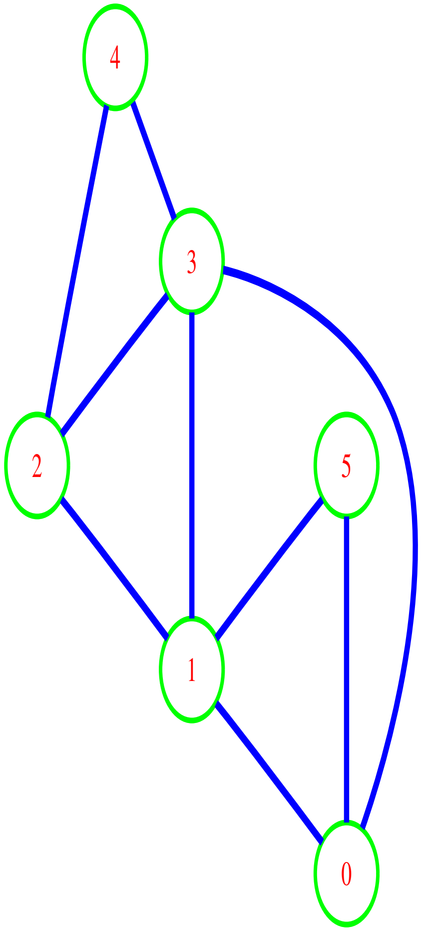

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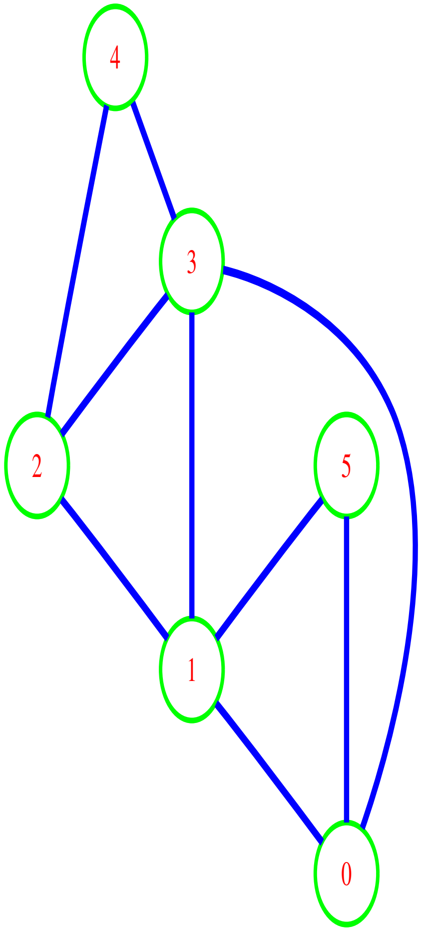

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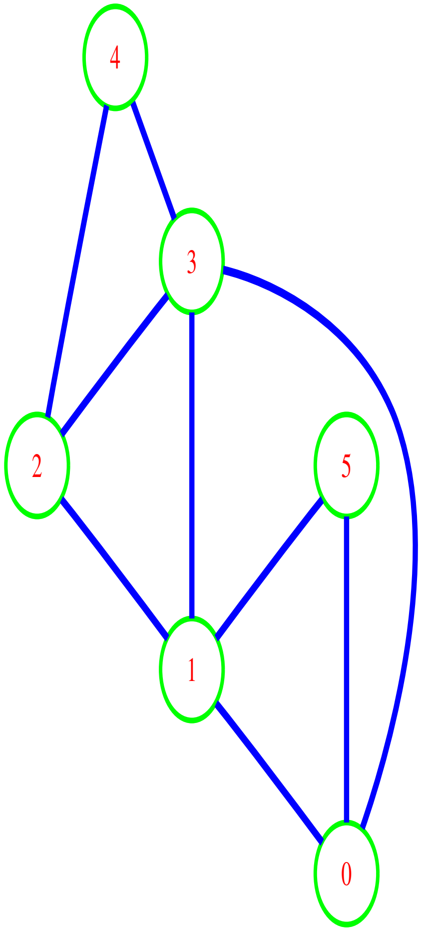

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Semidefinite programming and graph theory

Semidefinite programming (SDP) is a kind of generalized linear programming.

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- The aim of the project is to see what performance one can achieve on real graph theory problems of the types that come up in network modelling.
- Such problems include the Lovasz theta number (a lower bound for the chromatic number), the maximal stable set problem, and the maximum cut problem.
- An important outcome of the project would be a determination of how the computation time scales with problem size. Geometric optimization problems could also be studied.

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- However, I have an application in signal processing, where only the approximate SVD is required, but it is required to rapidly update the factors after a small change in the matrix (perturbation theory, if you like).
- I have an idea how to do this, and the project would be to develop the idea and implement it in software.

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- This project would improve on this by using automatic differentiation (AD) techniques to compute the local linearizations.
- I have much experience in AD and can supply software for this step.

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- C++ programming is needed (as operator overloading is necessary).

5. Fast random selection

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  - *Select exactly some specified number* $n \leqslant N$ *of them*
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- This project would investigate these methods and check their efficiency in practice. There are applications to the generation of large random graphs.

6. Random sampling of set partitions

- A partition of $[n] = \{1, 2, ..., n\}$ is an assignment each element to a subset called a block, without regard to the labelling of the block, or order of elements within a block. Thus for [5], 01|2|34 and 2|10|43 are the same partition.

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- Beyond about $n = 15$, there are too many partitions to allow us to construct all of them in reasonable time.
- Thus, for large sets, simulations must rely on a uniform random sampling procedure. This project would investigate such methods and produce a well-designed C library for use in other projects.

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- The algorithm should have practical applications in informatics; for example, counting the number of different packet types in a network.

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- The resulting algorithms generate in an unbiased manner discrete configurations that may have nontrivial symmetries, and they do so by means of real-arithmetic computations.
- This project would develop software for some of these methods, and measure its efficiency in practice. There are important applications in statistical computing.

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- This project would survey the field of convex optimization and investigate and evaluate this software. We would hope to try applications in the field of radio technology.

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- The application is to the historical study of ancient defence systems.

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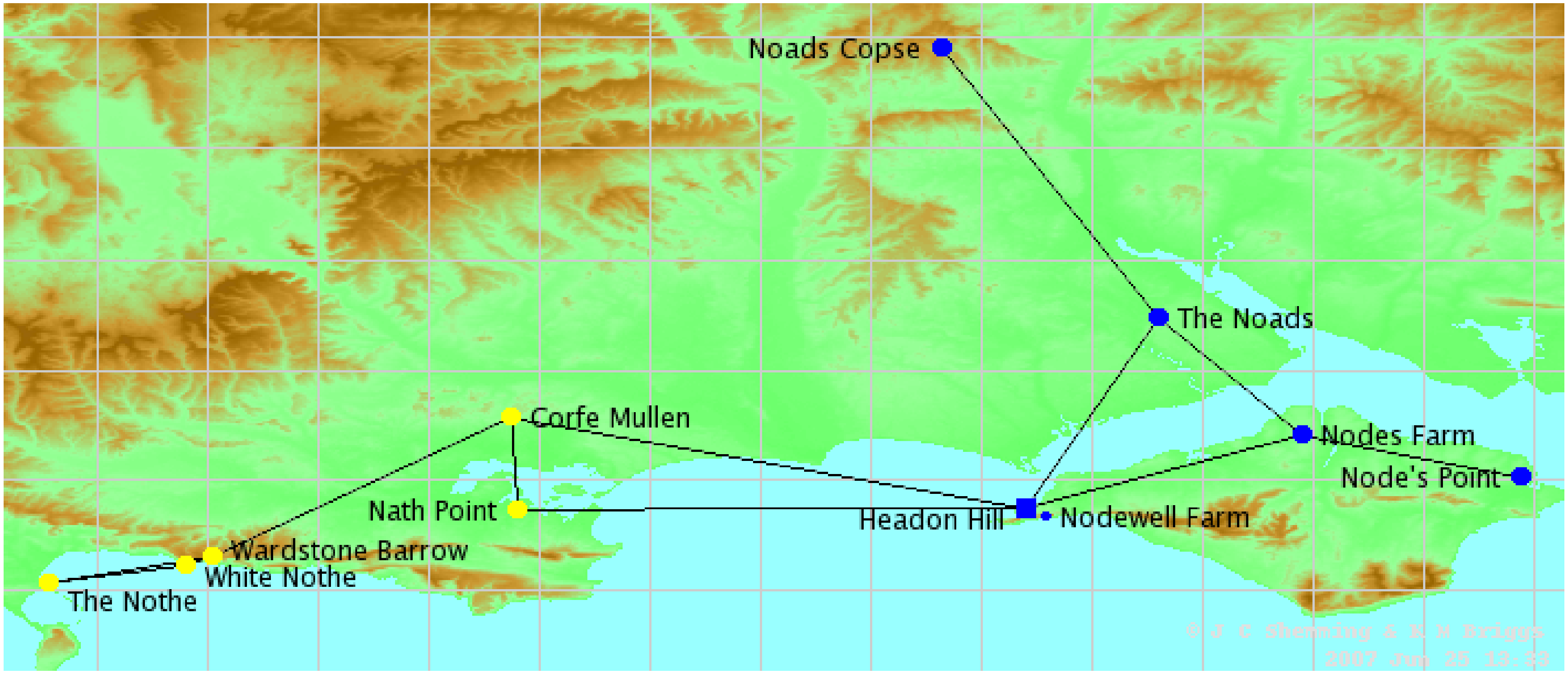

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- The maths involved is interesting computational geometry, and fast algorithms are needed for things like the point-in-polygon test, and for distance to the nearest of a given finite set of straight line segments.

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