Modelling train delays

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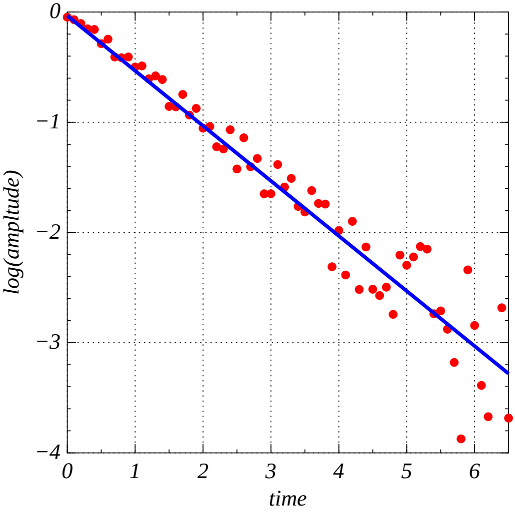
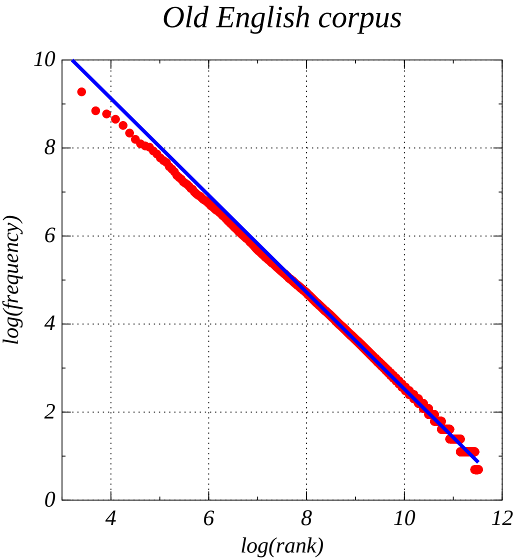
Exponential decay law

$\log$(amplitude) *vs.* time is a straight line.



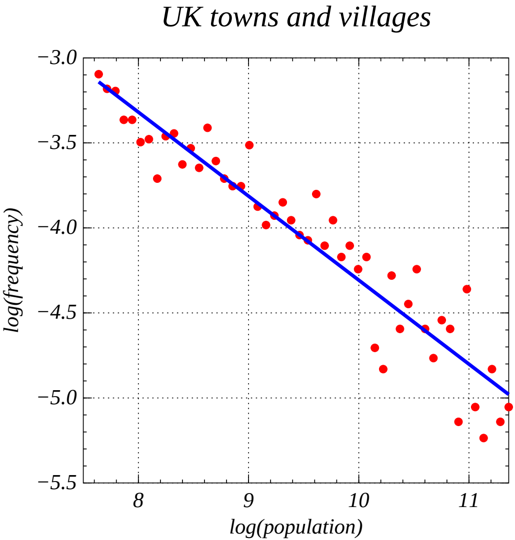
Zipf's law - OE corpus

$\log$(frequency) *vs.* $\log$(rank) is a straight line.



Population distribution

$\log(\text{frequency})$ *vs.* $\log(\text{population})$ is a straight line.



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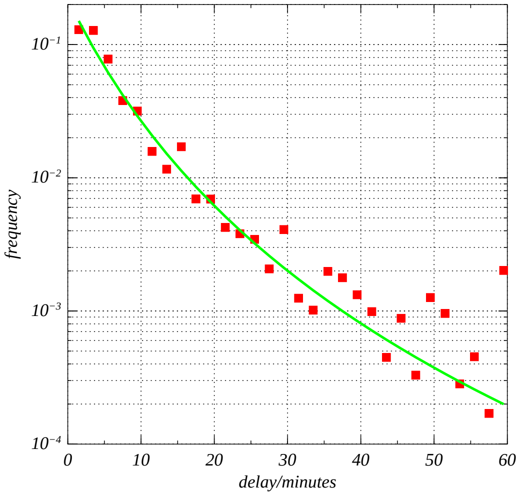
- $\lim_{q\to 1} e_{q,\beta}(t) = \exp(-\beta t)$
- large q gives a power-law (long tail)

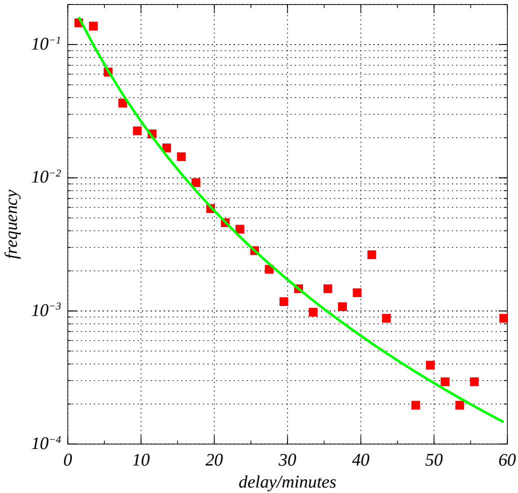
Manchester, all departures

$q = 1.207 \pm 0.0028$,
$\beta = 0.265 \pm 8.8e{-}07$.



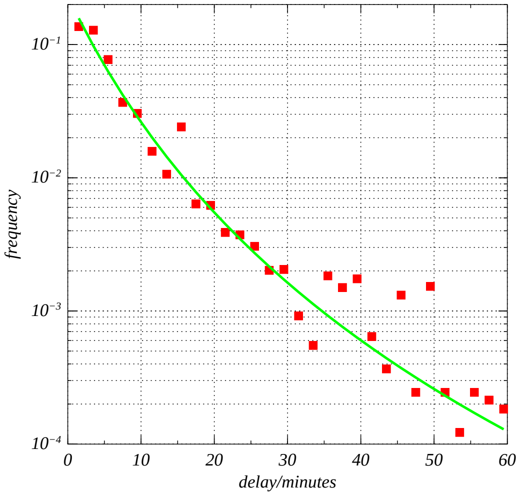
Manchester to Liverpool

$q = 1.19$
± 0.012,
$\beta = 0.273$
$\pm 2.4e-06$.



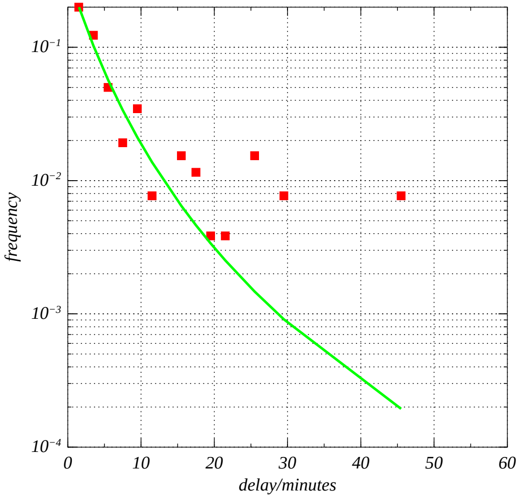
Manchester to airport

$q = 1.181 \pm 0.006$, $\beta = 0.268 \pm 1.3e{-}06$.

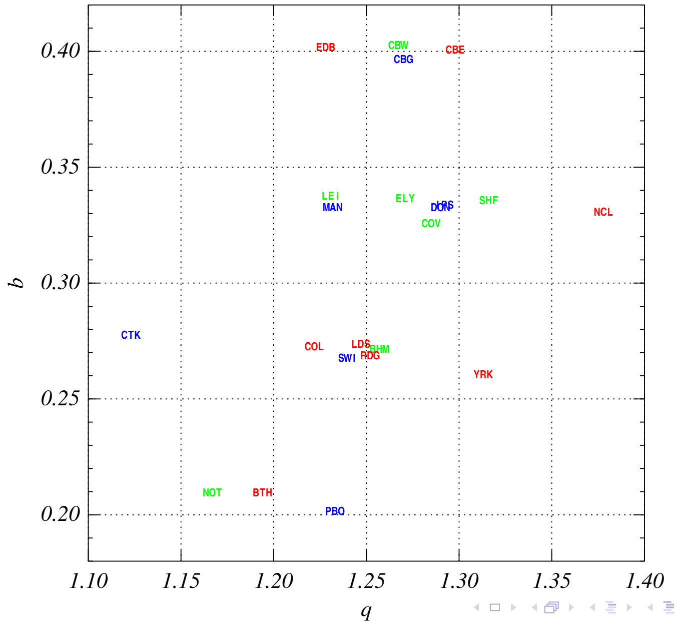


Manchester to London Euston

q=1.214 $\pm$0.097, β = 0.411 $\pm$ 0.00014.



Estimated parameters



Reference

K M Briggs & C Beck, *Modelling train delays with q-exponential functions* Physica **A 378**, 498–504 (2007).