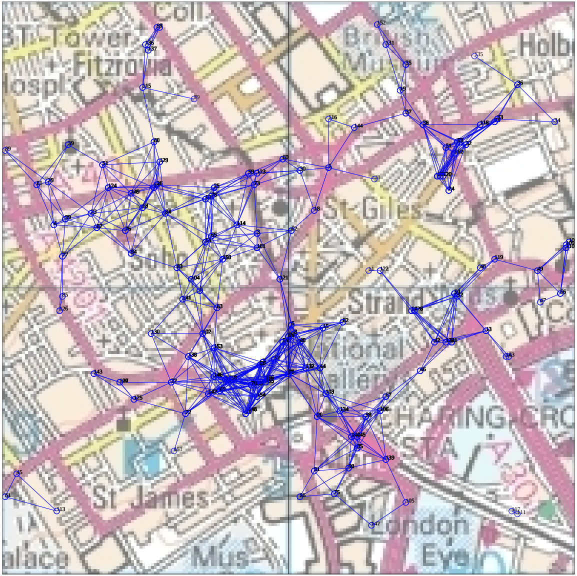
Graph models of wireless networks

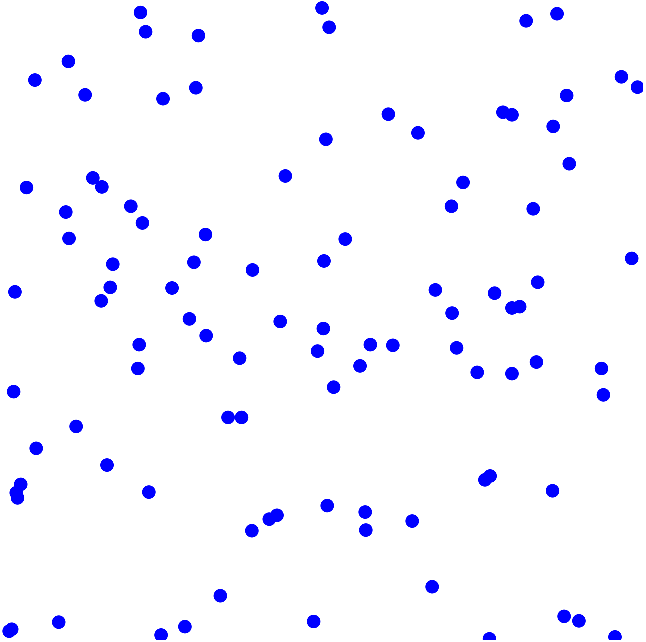
Keith Briggs
keith.briggs@bt.com

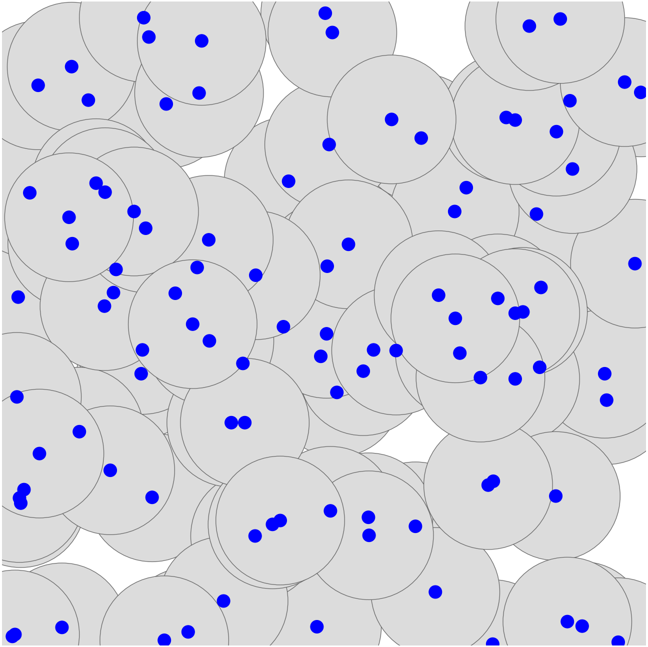
Complexity Research Group
BT Mobility Research Centre
http://keithbriggs.info

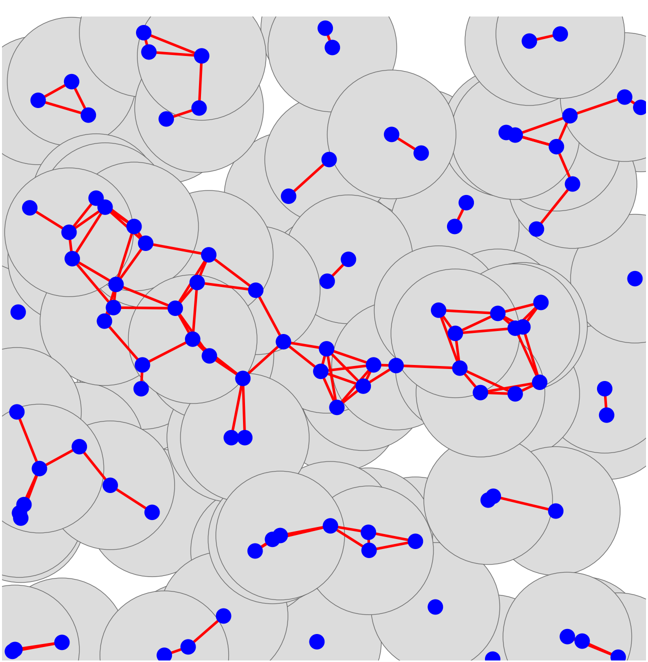
NET-ACE Brunel University 2008 Sep 19 0930

Wireless networks









PPPP(λ): definitions and statistical properties

- PPPP(λ): planar Poisson point process

PPPP(λ): definitions and statistical properties

- PPPP(λ): planar Poisson point process
  - region R has Poi($\lambda\|R\|$) points

PPPP(λ): definitions and statistical properties

- PPPP(λ): planar Poisson point process
  - region R has Poi($\lambda\|R\|$) points
  - numbers in non-overlapping regions are *independent*

PPPP(λ): definitions and statistical properties

- PPPP(λ): planar Poisson point process
  - region R has Poi($\lambda\|R\|$) points
  - numbers in non-overlapping regions are *independent*
  - pdf of distance X to kth nearest neighbour ($k{=}1,2,3,\ldots$) is (Haenggi)

$$f_k(x) = \frac{2(\lambda\pi)^k}{\Gamma(k)} \exp(-\lambda\pi x^2) x^{2k-1}$$

PPPP(λ): definitions and statistical properties

- PPPP(λ): planar Poisson point process
  - region R has Poi($\lambda \| R \|$) points
  - numbers in non-overlapping regions are *independent*
  - pdf of distance X to kth nearest neighbour ($k = 1, 2, 3, \ldots$) is (Haenggi)

$$f_k(x) = \frac{2(\lambda\pi)^k}{\Gamma(k)} \exp(-\lambda\pi x^2) x^{2k-1}$$

  - mean distances are $\frac{1}{2}\lambda^{-1/2}, \frac{3}{4}\lambda^{-1/2}, \frac{15}{16}\lambda^{-1/2}, \ldots$

PPPP(λ): definitions and statistical properties

- PPPP(λ): planar Poisson point process
  - region R has Poi($\lambda \| R \|$) points
  - numbers in non-overlapping regions are *independent*
  - pdf of distance X to kth nearest neighbour ($k = 1, 2, 3, \ldots$) is (Haenggi)

$$f_k(x) = \frac{2(\lambda\pi)^k}{\Gamma(k)} \exp(-\lambda\pi x^2) x^{2k-1}$$

  - mean distances are $\frac{1}{2}\lambda^{-1/2}, \frac{3}{4}\lambda^{-1/2}, \frac{15}{16}\lambda^{-1/2}, \ldots$
- also binomial PP
- also nonhomogeneous case

PPPP(λ): radial generation

- $s = 0$
- do

  $s \leftarrow s - \log(\text{Uniform}(0,1))$
  $\theta = 2\pi\text{Uniform}(0,1)$
  $r = \sqrt{s/(\pi\lambda)}$
  $x = r\cos\theta$
  $y = r\sin\theta$

- while $r <$ desired maximum radius

GRG(λ, ρ): definitions and statistical properties

- take PPPP(λ); call them *nodes*

GRG(λ, ρ): definitions and statistical properties

- take PPPP(λ); call them *nodes*
- connect nodes separated by less than ρ by *links* or *edges*

GRG(λ, ρ): definitions and statistical properties

- take PPPP(λ); call them *nodes*
- connect nodes separated by less than ρ by *links* or *edges*
- degree of a point (not necessarily a node) is defined as the number of Poisson points in its disk

GRG(λ, ρ): definitions and statistical properties

- take PPPP(λ); call them *nodes*
- connect nodes separated by less than ρ by *links* or *edges*
- degree of a point (not necessarily a node) is defined as the number of Poisson points in its disk
- distribution of degree is Poisson

GRG(λ, ρ): definitions and statistical properties

- take PPPP(λ); call them *nodes*
- connect nodes separated by less than ρ by *links* or *edges*
- degree of a point (not necessarily a node) is defined as the number of Poisson points in its disk
- distribution of degree is Poisson
- *degree* (D_0, D_1) *of an edge* is the degree of its end nodes

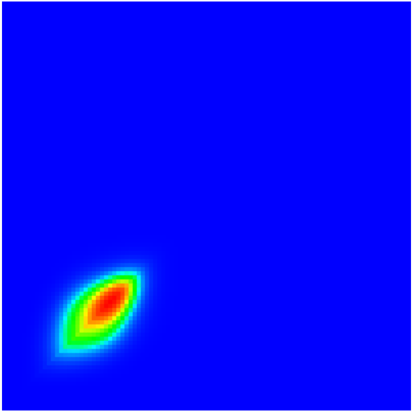
GRG(λ, ρ): definitions and statistical properties

- take PPPP(λ); call them *nodes*
- connect nodes separated by less than ρ by *links* or *edges*
- degree of a point (not necessarily a node) is defined as the number of Poisson points in its disk
- distribution of degree is Poisson
- *degree* (D_0, D_1) *of an edge* is the degree of its end nodes
- the cluster coefficient is $1 - \frac{3\sqrt{3}}{4\pi}$

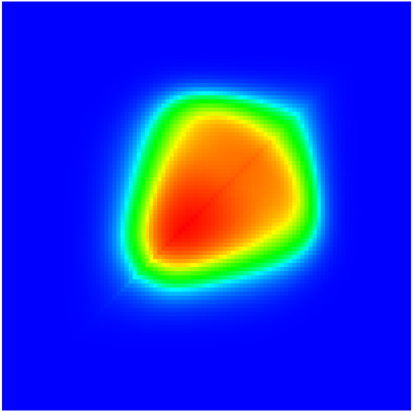
GRG(λ, ρ): definitions and statistical properties

- take PPPP(λ); call them *nodes*
- connect nodes separated by less than ρ by *links* or *edges*
- degree of a point (not necessarily a node) is defined as the number of Poisson points in its disk
- distribution of degree is Poisson
- *degree* (D_0, D_1) *of an edge* is the degree of its end nodes
- the cluster coefficient is $1 - \frac{3\sqrt{3}}{4\pi}$
- surprisingly, the degree-degree correlation is the same, independent of λ and ρ !

GRG(20, ρ) degree-degree distribution

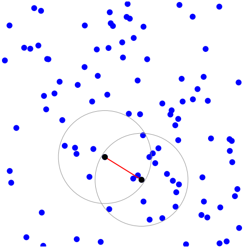


ρ=0.3



ρ=0.5

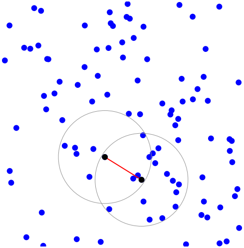
GRG(λ, ρ) degree-degree correlation



- If $X_0 \sim \text{Poi}(\lambda_0)$, $X_1 \sim \text{Poi}(\lambda_1)$, $X_2 \sim \text{Poi}(\lambda_1)$ are independent, and $Y_1 = X_1 + X_0$, $Y_2 = X_2 + X_0$, then

$$\text{corr}(Y_1, Y_2) = \frac{\lambda_0}{\lambda_0 + \lambda_1}$$

GRG(λ, ρ) degree-degree correlation



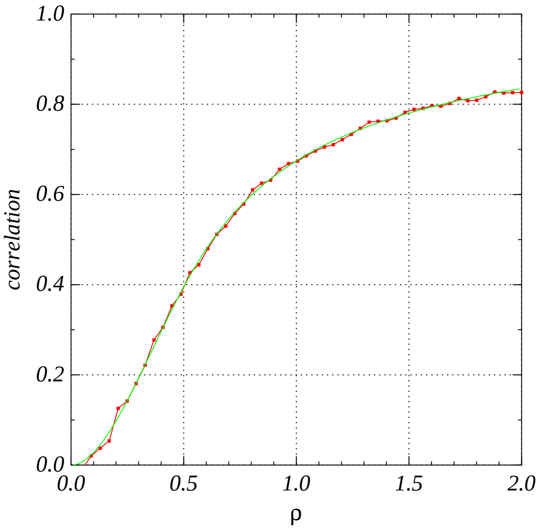
- If $X_0 \sim \text{Poi}(\lambda_0)$, $X_1 \sim \text{Poi}(\lambda_1)$, $X_2 \sim \text{Poi}(\lambda_1)$ are independent, and $Y_1 = X_1 + X_0$, $Y_2 = X_2 + X_0$, then

$$\text{corr}(Y_1, Y_2) = \frac{\lambda_0}{\lambda_0 + \lambda_1}$$

- For PPPP, degree-degree correlation is E[corr]

$$= \int_0^\rho \frac{2\rho^2 \arccos(x/(2\rho)) - (x/2)\sqrt{4\rho^2 - x^2}}{\pi\rho^2} \frac{2x}{\rho^2} \, \mathrm{d}x$$
$$= 1 - 3\sqrt{3}/(4\pi) \simeq 0.5865$$

GRG(λ, ρ) degree-degree correlation - square



exact (doable but messy); simulation

GRG($\lambda, \rho,$ unit circle): degree distribution

- pdf of distance of a random point from the centre, given that it is within $1-\rho$ of the edge:

$$f_\rho(x) = \frac{(4-2\rho)x + 2\rho - 2}{\rho^3 - 2\rho^2 + 2\rho} [\![1-\rho < x < 1]\!]$$

GRG(λ, ρ, unit circle): degree distribution

- pdf of distance of a random point from the centre, given that it is within $1-\rho$ of the edge:

$$f_\rho(x) = \frac{(4-2\rho)x + 2\rho - 2}{\rho^3 - 2\rho^2 + 2\rho} [\![1-\rho < x < 1]\!]$$

- area of overlap of circles radius 1 and ρ, centres x apart:

$$A(x) = \rho^2 \arccos\left(\frac{x^2+\rho^2-1}{2x\rho}\right) + \arccos\left(\frac{x^2-\rho^2+1}{2x}\right) - \frac{1}{2}[(1-x+\rho)(x+\rho-1)(x-\rho+1)(x+\rho+1)]^{1/2}$$

GRG(λ, ρ, unit circle): degree distribution

- pdf of distance of a random point from the centre, given that it is within $1-\rho$ of the edge:

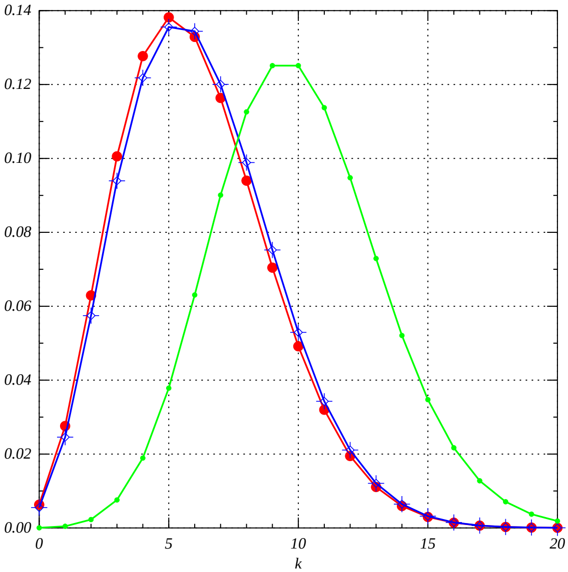
$$f_\rho(x) = \frac{(4-2\rho)x + 2\rho - 2}{\rho^3 - 2\rho^2 + 2\rho} [\![1-\rho < x < 1]\!]$$

- area of overlap of circles radius 1 and ρ, centres x apart:

$$A(x) = \rho^2 \arccos\left(\frac{x^2+\rho^2-1}{2x\rho}\right) + \arccos\left(\frac{x^2-\rho^2+1}{2x}\right) - \frac{1}{2}[(1-x+\rho)(x+\rho-1)(x-\rho+1)(x+\rho+1)]^{1/2}$$

- Prob$[d=k] =$ $(1-\rho)^2\,\mathrm{Poi}(A(x),k) + \rho(2-\rho)\int_{1-\rho}^{1} \mathrm{Poi}(A(x)\lambda)\, f_\rho(x)\,\mathrm{d}x$
- where $\mathrm{Poi}(\mu,k) = e^{-\mu}\mu^k/k!$

GRG(λ, ρ, unit circle) degree distribution



exact; simulation;
Poisson - ignores
edge effect.

Poisson maxima 1

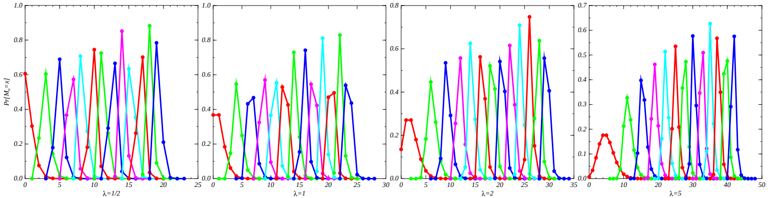
- $\{X_1, X_2, \ldots, X_n\}$ iid, $\Pr[X_i = k] = e^{-\lambda}\lambda^k/k!$

Poisson maxima 1

- $\{X_1, X_2, \ldots, X_n\}$ iid, $\Pr[X_i = k] = e^{-\lambda}\lambda^k/k!$
- $M_n = \max(X_i)$

Poisson maxima 1

- $\{X_1, X_2, \ldots, X_n\}$ iid, $\Pr[X_i{=}k]{=}e^{-\lambda}\lambda^k/k!$
- $M_n{=}\max(X_i)$



The distribution of the maximum of Poisson variables for $\lambda{=}1/2, 1, 2, 5$ (left to right) and $n{=}10^0, 10^2, 10^4, \ldots, 10^{24}$

Poisson maxima 2

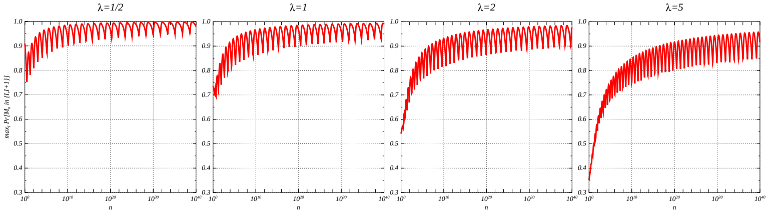
- Anderson: $\exists I_n \in \mathbb{Z}$ s.t. $\Pr M_n \in (I_n, I_{n+1}) \to 1$

Poisson maxima 2

- Anderson: $\exists I_n \in \mathbb{Z}$ s.t. $\Pr M_n \in (I_n, I_{n+1}) \to 1$
- Kimber: $I_n \sim \log n / \log \log n$ as $n \to \infty$

Poisson maxima 2

- Anderson: $\exists I_n \in \mathbb{Z}$ s.t. $\Pr M_n \in (I_n, I_{n+1}) \to 1$
- Kimber: $I_n \sim \log n / \log\log n$ as $n \to \infty$



The maximal probability (with respect to I_n) that $M_n \in \{I_n, I_n+1\}$ for $\lambda = 1/2, 1, 2, 5$ (left to right) and $10^0 \leqslant n \leqslant 10^{40}$. The curves show the probability that M_n takes either of its two most frequently occurring values.

Poisson maxima 3

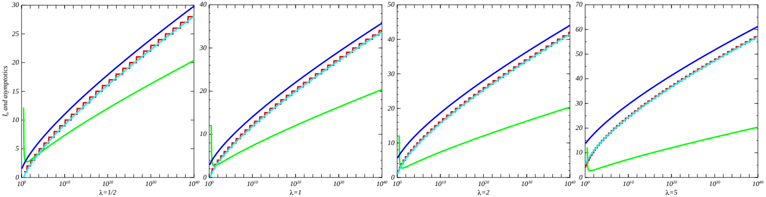
- $M_n \sim x_0 \equiv \log n / W\left(\frac{\log n}{e\,\lambda}\right)$

Poisson maxima 3

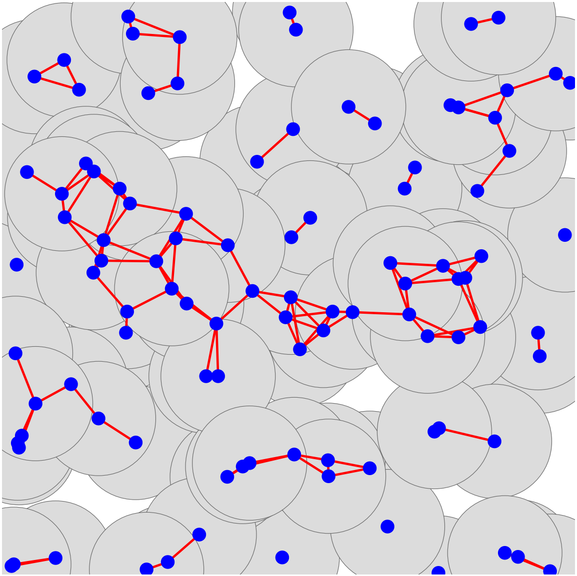
- $M_n \sim x_0 \equiv \log n / W\left(\frac{\log n}{e\,\lambda}\right)$
- $M_n \sim x_1 = x_0 + \frac{\log \lambda - \lambda - \log(2\pi)/2 - 3\log(x_0)/2}{\log(x_0) - \log \lambda}$

Poisson maxima 3

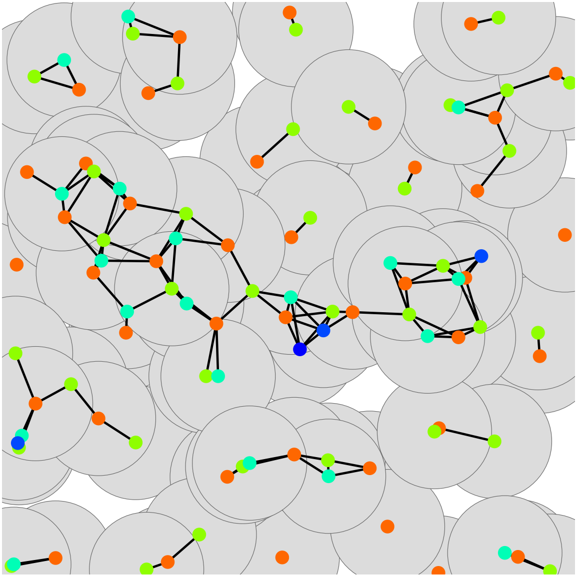
- $M_n \sim x_0 \equiv \log n / W\left(\frac{\log n}{e\,\lambda}\right)$
- $M_n \sim x_1 = x_0 + \frac{\log\lambda - \lambda - \log(2\pi)/2 - 3\log(x_0)/2}{\log(x_0) - \log\lambda}$



Chromatic number



Chromatic number



Objectives

- Optimization problem:

$$\min_x f(x)$$

- Minimizing the average interference:

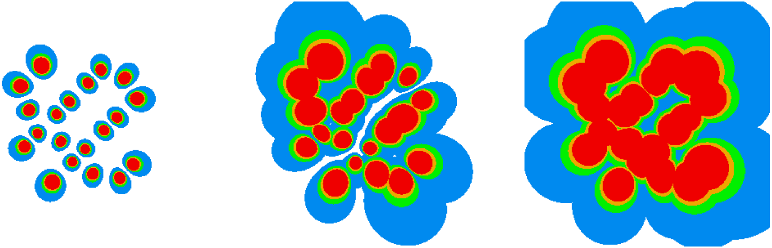
$$f(x) = \frac{1}{n} \sum_i I_i(x)$$

- Minimizing the maximum interference:

$$f(x) = \max_i I_i(x)$$

Optimization results

Modulation schemes: 11Mbps, 5.5Mbps, 2Mbps, 1Mbps



(a) 20 APs using the same power level and channel

(b) 20 APs with randomly assigned channels

(c) 20 APs using the same power level, but with an optimized channel allocation (13 channels)