The maths of train delays

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Exponential decay law

\[ \log(\text{amplitude}) \text{ vs. time is a straight line.} \]
Zipf’s law

log(frequency) vs. log(rank) is a straight line.

Old English corpus
Population distribution

*UK towns and villages*

\[ \log(\text{frequency}) \quad \text{vs.} \quad \log(\text{population}) \quad \text{is a straight line.} \]
The \( q \)-exponential law

\[
\begin{align*}
\textbullet \quad e_{q \beta}(x) & := (1 + \beta(q-1)x)^{1/(1-q)} \\
\lim_{q \to 1} e_{q \beta}(t) & = \exp(-\beta t)
\end{align*}
\]
The $q$–exponential law

- $e_{q,\beta}(x) := (1 + \beta(q - 1)x)^{1/(1-q)}$

- $\lim_{q \to 1} e_{q,\beta}(t) = \exp(-\beta t)$
The $q$–exponential law

- $e_{q,\beta}(x) := (1 + \beta(q - 1)x)^{1/(1-q)}$
- $\lim_{q \to 1} e_{q,\beta}(t) = \exp(-\beta t)$
- Large $q$ gives a power-law (long tail)
Ipswich to London

Exponential laws

Power laws

$q$—exponential laws

How good is it?
Ipswich to Norwich

Exponential laws

Power laws

$q$-exponential laws

How good is it?
Newcastle to Edinburgh

Exponential laws
Power laws
q—exponential laws

How good is it?