Modelling internet round-trip time data

Keith Briggs

Keith.Briggs@bt.com

http://research.btexact.com/teralab/keithbriggs.html

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Outline

- motivation
- data
- theory
- model fitting
Motivation

- internet as a complex system
- round-trip time (RTT) data forms an intriguing time series
- successful models would allow:
  - forecasting
  - simulation
  - understanding

- any model used should incorporate features believed to exist in the data in a *natural* way
100 days of typical raw data, from www.edinburgh.ac.uk
Raw data 2

www.edinburgh.ac.uk and www.chem.uwa.edu.au
Long-range dependence?

- definition:
  \[
  \lim_{k \to \infty} \rho(k) \, k^{2(1-H)} = \text{constant}
  \]

- \(0 < \alpha = 2(1-H) < 1\)

- useless!
  - for stationary process only
  - large-k limit
  - \(H\) cannot be estimated in practice even when it exists and is known
  - tries to reduce complex phenomena to a single number
Wavelet transform 1

- I use the Haar basis - left: scaling function $\phi$; right: wavelet function $\psi$

\[
\psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k)
\]

\[
\phi_{j,k}(t) = 2^{j/2} \phi(2^j t - k)
\]
Wavelet transform 2

\[ f_t = \sum_k U_{0,k} \phi_{0,k}(t) + \sum_{j=0}^{J} \sum_k W_{j,k} \psi_{j,k}(t) \]

wavelet coefficients, \( W_{j,k} \), and scaling coefficients, \( U_{j,k} \), are defined by

\[ W_{j,k} = \sum_{t=1}^{T} f_t \psi_{j,k}(t) \]
\[ U_{j,k} = \sum_{t=1}^{T} f_t \phi_{j,k}(t) \]
Example wavelet transform 2

Information at scales $J$, $J-1$ and $J-4$
Multifractal spectrum

- $g$ is Lipschitz $\alpha$ at $x_0$ if $\alpha$ is the supremum of those $a$ such that in a neighbourhood of $x_0$

\[
\left| g(x) - p_{\lfloor \beta \rfloor}(x) \right| = \mathcal{O}(\left| x - x_0 \right|^a)
\]

where $p_{\lfloor \alpha \rfloor}$ is a polynomial of degree $\lfloor \alpha \rfloor$.

- Let $f(\alpha)$ be the Hausdorff dimension of the set of points where the Lipschitz exponent is $\alpha$. This is the **multifractal spectrum** of $g$.

- example:
  - fractal Brownian motion has zero mean and Gaussian increments s. t. the mean square increment at lag $\Delta$ is proportional to $|\Delta|^{2H}$
  - this is a monofractal - $f(\alpha) = \delta(H)$
  - however, estimates of $f$ from a finite sample will not show this delta function behaviour.
The multifractal formalism

- to estimate \( f(\alpha) \) from discrete data, use partition function \( \tau(q) \)

- For a process \( X(t) \), define the **structure function**, \( S_j(q) \), by

\[
S_j(q) = \sum_{k_j} (2^{-j/2} U_{j,k_j})^q
\]

where \( U_{j,k_j} \) are Haar scaling coefficients for \( X(t) \)

- The partition function is then defined as

\[
\tau(q) = - \lim_{j \to \infty} \frac{1}{j} \log S_j(q)
\]

- Next define

\[
f_L(\alpha) = \inf_{q \in \mathbb{R}} (q\alpha - \tau(q))
\]
The multifractal formalism shows that

\[ f(\alpha) \leq f_L(\alpha) \]

The Legendre transform of the partition function is the concave hull of \( f(\alpha) \). \( f_L(\alpha) \) is known as the \textit{Legendre spectrum}.

Let us assume that our RTT data is a sample of an underlying continuous process. Assume further that the observable scaling behaviour of \( S_j(q) \) is continued beyond the finest measured scale to the limit \( j \to \infty \). That is,

\[ S_j(q) \approx 2^{-j\tau(q)} \]

over \( j = j_1, \ldots, j_2 \), where \( j_1, j_2 \in [0, J] \).

\( \tau(q) \) can be estimated from the gradient of a plot of \( \log_2 S_j(q) \) against \( j \) over a finite range of scales.
Riedi's multifractal wavelet model

- Riedi and others have proposed a stochastic variant of this type of multifractal model.
- We let the wavelet coefficients from one scale to the next be iid random variables. For example, from the Beta distribution.
- Fitting to data then involves estimating the parameters in the distribution.
- Simulating involves drawing random variates from the fitted distribution.
- Determinism and preprocessing:
  - Always remove any clear deterministic features from the data first:
    - trends
    - periodic components
    - baseline shifts
Example data analysis

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<tr>
<th>Data</th>
<th>HMWM</th>
<th>βMWM</th>
</tr>
</thead>
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<td></td>
</tr>
<tr>
<td>0.98</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

HMWM, βMWM and www.chem.uwa.edu.au multifractal spectra
2^{12} points from www.chem.uwa.edu.au (top) compared with a $\beta$MWM realisation (bottom)
www.chem.uwa.edu.au, HMWM, and $\beta$MWM autocorrelation
Conclusion and references

- There are many open questions in this type of work and few concrete models.
- Fitting and parameter estimation is a major problem.
- Full report and bibliography:
  - Analysis and simulation of internet round-trip times