Optimizing antenna beamforming with quantum computing

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***Abstract*—We consider beamforming with large-scale antenna arrays in which the elements can transmit only in one of a small number of phase-shifts. This creates an NP-hard optimization problem, namely the maximization of the ratio of two Hermitian quadratic forms, with the state vector constrained to the set of allowed phase-shifts. We show how the maximization problem can be rigorously solved by reformulating it as a sequence of quadratic (but non-convex) minimization problems. These minimization problems can be solved exactly with integer linear programming when sufficiently small. When they are large there is no good classical solution method, but we show that they can be solved using quantum computers of the annealing type. Not only is there a large improvement in solution time with quantum computing, there is also the potential for energy saving.**

***Index Terms*—Multi-element antenna arrays, MIMO, beamforming, quantum computing.**

I. Introduction

In this paper, we consider an abstraction of an antenna array as a collection (not necessarily planar) of isotropically radiating point elements. These might represent active elements fed from cables, or passive elements used in either reflective or transmissive mode. Our inspiration is the recent use of PIN diode arrays for reflecting or focussing millimetre-wave beams. These are currently opening up a cheap, simple, and reliable new technology with important new applications in propagating millimetre-wave signals beyond the usual line-of-sight restrictions.

II. Mathematical formulation

The computation of the far-field signal generated by an array of radiating elements is standard; see, for example, Chapter 3 in [1]. For beamforming, it is necessary to define an optimization problem with an appropriate objective function. A good option is to define a small connected region R on the surface of a large sphere surrounding the array, and try to maximize the ratio of energy flux through R to energy flux over the whole sphere S (or, optionally, not through R). Note that this does not explicitly constrain the undesired side-lobes.

We base our work on the formulation of Oliveri et al. [2], but extend it fully into three dimensions (rather than just a half-space), and also choose R to be a circular patch, rather than the vertical projection of a planar square patch onto the sphere as used by Oliveri et al. This formulation results in a maximization problem of the type

$$\max_{w} \frac{w^H A w}{w^H B w}. \tag{1}$$

Here w is a complex vector, and A and B are Hermitian positive-semidefinite matrices defined by surface integrals over R and S respectively. A and B are functions of the array geometry, beam azimuth and elevation, and beam angular diameter. If w is unconstrained, this maximization problem can be easily solved by finding the largest eigenvalue $\lambda_{\max}$ of the generalized eigenvalue problem $Aw=\lambda Bw$, and the corresponding eigenvector $w_{\max}$ is the desired steering vector. A typical solution obtained this way is shown in Figure 1 (left).

Suppose now that the elements of w can take only a small number (we consider only two or four) of fixed, pre-specified values. We will focus on the case in which the fixed values have modulus one, and thus refer to the 2-phase or 4-phase cases. This restriction models the case of PIN diodes mentioned above. One PIN diode creates two possible phase shifts, and a layer of two diodes creates four phase shifts. We also optionally allow a perfectly reflecting backplane, which is modelled by placing virtual elements on

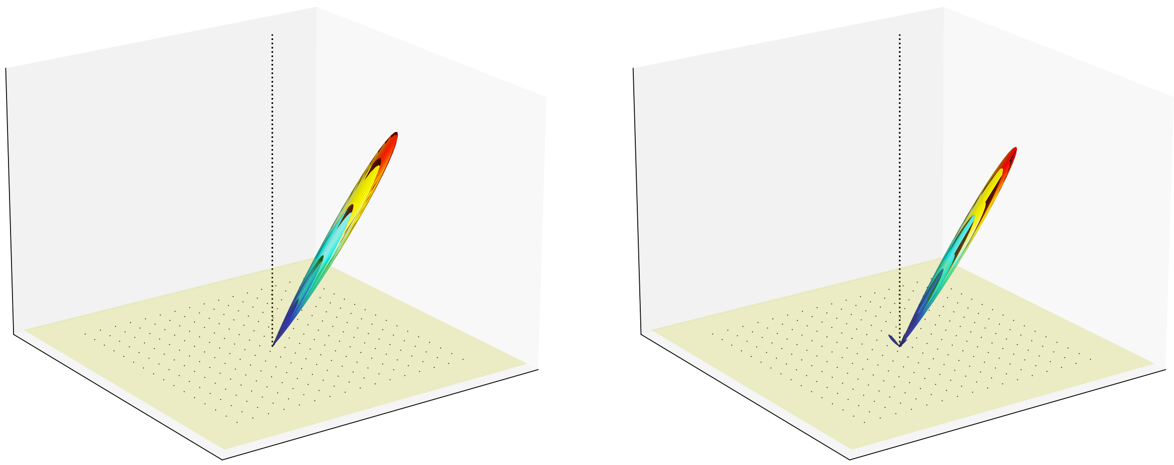

Fig. 1. (left) Beamforming using an $16{\times}16$ array of continuously weighted elements, with backplane; exact solution via eigenvalues. The elements are spaced a half-wavelength apart, and are situated in the $x{-}y$ plane, with the backplane (yellow) a quarter of a wavelength below. The beam polar angle is 0.7 radians, relative to the vertical dotted line (z-axis), and the beam azimuth is 2. The spectral colours roughly represent the gain, which in this case peaks at 27.1dB on the boresight, with the peak shown in red. Because the figure is a 3d projection onto 2d with transparency of the beam, an exact mapping of colours to gain is not possible. (right) The same problem with the weights constrained to the four constant-amplitude phase-shifts $\{1, j, -1, -j\}$; approximate solution with Metropolis heuristic. The peak gain is only slightly reduced to 25.1dB, but the beamwidth is similar. The colour scale is the same as in the left figure.

the opposite side of the plane to the real elements. Solving (1) under such constraints is now an NP-hard combinatorial optimization problem. An equivalent integer linear programming problem can be written down, but only solved in practice with a maximum number of array elements of about 40, even in the 2-phase case. Experience shows that computation times grows exponentially with array size.

III. Solution technique

The objective in (1) is the ratio of two positive convex functions. The whole problem is quasiconvex, and as such a bisection method is applicable, as described in [3, p. 145]. Let us first consider a more general form, with positive convex functions $f(x), g(x)$, both mapping $\mathbb{R}^n$ to $\mathbb{R}$, and C some constraint set (typically $C{=}\{0,1\}^n$):

$$\max_{x \in C} \frac{f(x)}{g(x)}. \tag{2}$$

This is precisely equivalent to

$$\min_{t \in \mathbb{R}} t \quad \text{such that} \quad f(x){-}tg(x){<}0 \quad \forall\, x{\in}C. \tag{3}$$

Note that $f(x){-}tg(x)$ is non-convex, even though f and g are convex. To start the bisection, we first need to bound the optimal t. For the antenna problem, the initial bounds $(t_0, t_1){=}(0, 1)$ will always work; we will be defining $f(x){=}x^TAx$ and $g(x){=}x^TBx$ with A is defined by an integral over a smaller region than B, and so the optimal t cannot be greater than unity. One bisection step then involves setting a trial t half-way between current the bounds (t_0, t_1), and solving the minimization problem

$$\min_{x \in C} f(x){-}tg(x). \tag{4}$$

If the optimal value of expression (4) is positive, then it follows that the trial t is too small, so we can raise the lower bound to $t_0{=}t$. Conversely, if the optimal value is negative, we set $t_1{=}t$. Bisection can be stopped when the interval (t_0, t_1) is sufficiently small. Note that during the minimization process, the finding of *any* positive value for $f(x){-}tg(x)$ means that the constraint in (3) is violated, and this allows us to abort the minimization early and make the correct decision $t_0{=}t$. This means that a heuristic minimization method can be safely used for updating the lower bound. This is, however, not the case for upper bound updates, so these steps, whether done by integer linear programming or otherwise, are typically slower.

IV. Applying the solution technique to antenna arrays

The quantum computing technique we will be using requires the optimization variable x to be a bit-vector, containing only the values 0 or 1. We map these to complex weights by an affine map; in the 2-phase case this takes the form $w_i = \alpha x_i + \beta$ for appropriate complex constants α and β. In the 4-phase case the map is similar, but uses two bits for each element of the weight vector. The optimization problem now has the following slightly more general form for an antenna array with n elements, and the bisection method still applies. In this form all parameters ($a, c \in \mathbb{R}^n$, $b, d \in \mathbb{R}$), variables, and function values are now real:

$$\max_{x \in \{0,1\}^n} \frac{x^T A x + a^T x + b}{x^H B x + c^T x + d}. \tag{5}$$

For this objective, the specific form of (4), which forms the constraint in (2), is now

$$\min_{x \in \{0,1\}^n} x^T (A - tB) x + (a - tc)^T x + b - d. \tag{6}$$

Note that a linear term like $a^T x$ can be absorbed into the diagonal elements of A, since $x_i^2 = x_i$ for 0-1 vectors. The constant terms defined by b and d are simple shifts which make no essential difference to the solution procedure. Figure 1 (right) shows a solution obtained by this method.

V. Classical and quantum annealing

The possibility of using quantum computing now arises because quantum annealing machines such as D-Wave can solve quadratic minimization problems of the type in (6). Here we think of x as a vector of 'spins', and an expression such as $x^T A x$ as a total system energy. We try to find the ground (lowest energy) state of the system. An introduction to such techniques is in [4], and a recent evaluation of the state of the art is in [5]. This problem-type is often called QUBO, for 'quadratic Boolean optimization' [6].

However, before describing how we have implemented this, we point out that a approximate simulation of the D-Wave machine on a conventional computer is useful to confirm the correctness of the implementation, and in fact is reasonably effective in its own right as a solution method. This is simply the well-known Metropolis algorithm. It proceeds by picking one of the spins at random, and computing the change in total system energy if this bit were 'flipped' (that is, the values 0 and 1 are interchanged). If this energy changed is negative, the new bit value is accepted, and the step is complete. Otherwise, the flip is accepted with some small probability (say, 1%). This probably can be reduced over time, to simulate 'freezing' of the system.

VI. Results using D-Wave quantum annealer

Using both a simulated annealer and the D-Wave Advantage [7] Quantum Processing Unit (QPU), we could find solutions for much larger antenna arrays than via integer linear programming, which in the 4-phase case is very slow for antenna arrays with more than 5×5 elements.

For small problems, for which we can compute an exact discrete solution for comparison, we found that slight differences in the resulting weight vector had negligible effect on beam-width and gain. For problems too large to be solved exactly, we can therefore have confidence that this method finds solutions, which if not necessarily globally optimal, are still very adequate for practical use. Any deviation from the true optimal solution can be justified by the enormous time saving from using heuristic solution methods.

It is difficult to define meaningful comparisons of classical and quantum solution times. In the current state of the art, using the quantum computer typically requires far more time preprocessing the problem (on a conventional computer) in order to prepare it for the QPU, than actual solution time. Thus, the total time taken to solve large problems using D-Wave was very long, and was in fact comparable with the time taken to find an exact solution using integer linear programming. Solving for the weight vector which generated Figure 2 (right) took around 90 minutes, compared to the approximately 30 seconds needed classically. However, breaking down the timings for the D-Wave process, we see that for the beam in Figure 2 (right) each annealing step took no more than 20μs. This is much faster than would be possible classically; the same problem using simulated annealing on a typical desktop computer takes times of the order

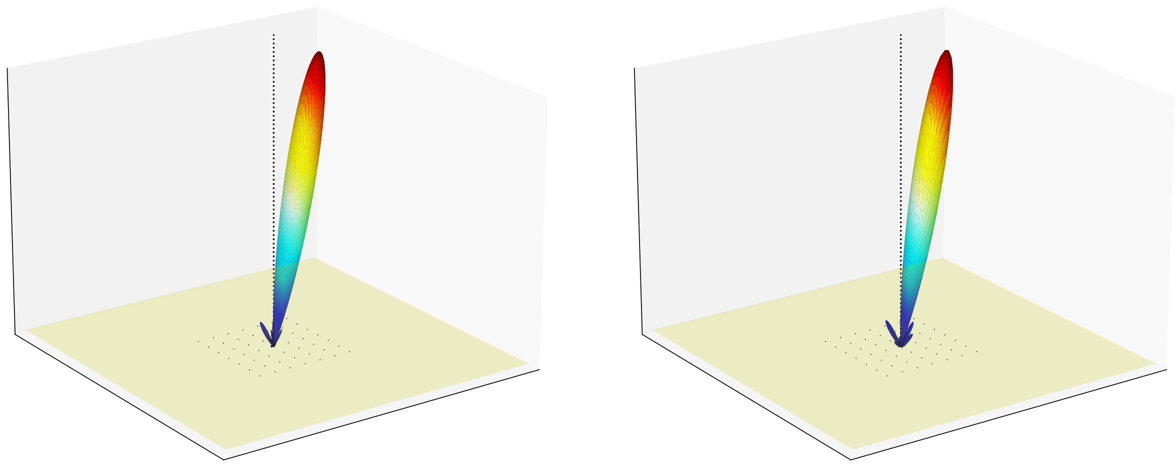

Fig. 2. (left) Beamforming using a 7×7 array of 4-phase elements, with backplane, azimuth=2, polar angle=0.25. The constant-amplitude phase-shifts were $\{1, j, -1, -j\}$, and the problem was solved by classical simulated annealing. (right) The same problem solved with the D-Wave quantum annealer. The small differences are due to the heuristic nature of the annealing process, meaning that slightly different (but still close to optimal) weights are found.

of seconds. However, at the current state-of-the-art in quantum annealing, these timing comparisons are not very meaningful. As well as timing improvement, there is a large potential for the energy consumed by the solution process to be reduced; and this is an area of much current research, in the BeGREEN project (acknowledged below) and elsewhere.

These experiences illustrate well a current problem with quantum annealing. Before D-Wave can begin an annealing process, each optimization variable needs to be 'embedded' (mapped) onto a qubit in the QPU. Embedding the minimization problem for each bisection step using the D-Wave automatic embedding function took a very long time, resulting in the significant increase to the overall run-time. It is likely that by using a manual embedding of the problem this time difference could be lessened. A potential line for further study would be to consider the embedding for each bisection step and look for patterns which could be exploited to minimize the number of embeddings that needed to be done. However, as the matrices resulting from this problem are dense, it is likely that solving this embedding problem will not be trivial. We conclude that further research is needed to make quantum annealing practical for large (say, 16×16 elements, 4-phase) antenna-array problems.

VII. Conclusion

We have shown how the fractional quadratic maximization problem arising in optimizing antennas beams can be reformulated as a sequence of quadratic minimization problems for which several solution methods exist. Most importantly, the quadratic minimization problems are of the type directly solvable on quantum annealers. The main novelty of this paper is showing that this reformulation is a transformation into an exactly equivalent sequence of optimization problems, and it puts the problem into a form ideally suited to the quantum annealer. This method is shown to work well in principle and provides a significant improvement on the size of problems which can be solved, compared to exact solution methods. However, with the presently available D-Wave technology, the embedding problem is a serious bottleneck for large problems and awaits better solution methods.

Acknowledgment

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