

Delayed iteration

Keith Briggs

Keith.Briggs@bt.com

`more.btexact.com/people/briggsk2`

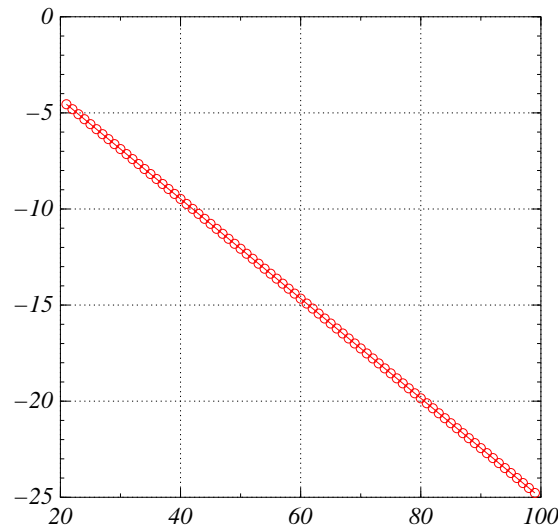
CRG meeting 2004 July 05 1500

`sodium:tex/delayediteration2004jul05.tex`

TYPESET 2004 JULY 5 14:18 IN PDF \LaTeX ON A LINUX SYSTEM

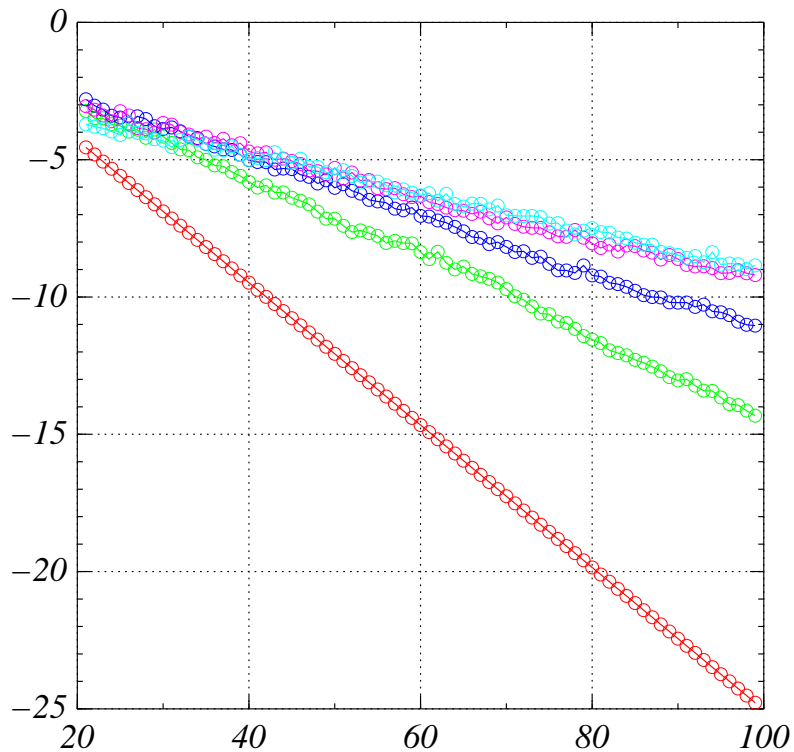
Normal iteration

- ★ $A, \rho(A) \equiv |\lambda_{\max}| < 1$
- ★ $x(i+1) = A x(i), \quad i = 0, 1, 2, \dots$
- ★ $\|x(i)\| \rightarrow 0$
- ★ $\lim_{i \rightarrow \infty} \frac{1}{i} \log \|x(i)\|$ exists
- ★ call this the convergence rate
- ★ convergence time = $1/(\text{convergence rate})$



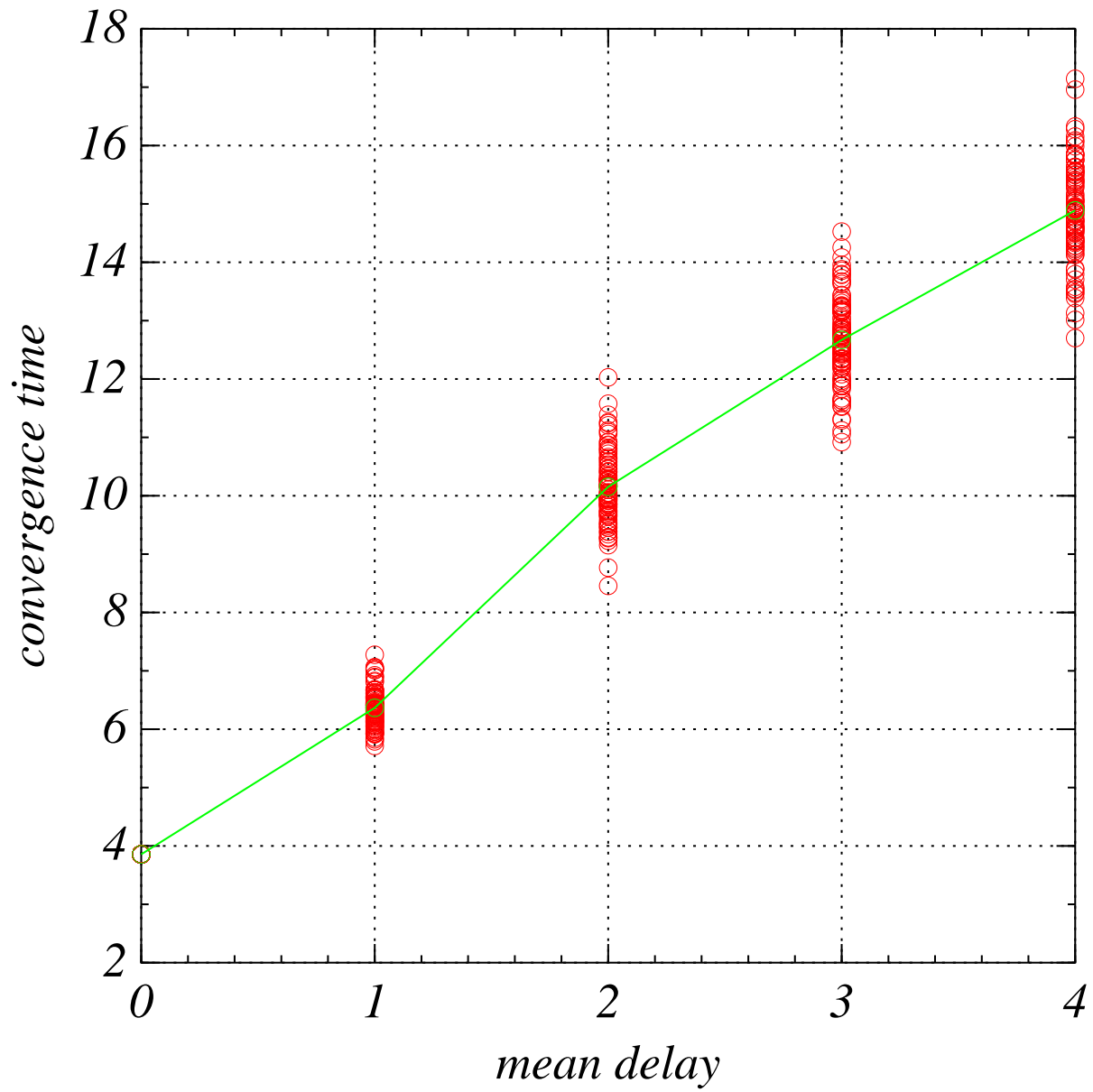
Delayed iteration 1

- ★ Suppose each component of x is out-of-date by an exponentially distributed time.
- ★ Then we expect convergence to be slower, but how much slower?



$$A = \begin{bmatrix} 0.5 & 0.2 & 0.0 & 0.1 \\ 0.2 & 0.4 & 0.2 & 0.2 \\ 0.0 & 0.2 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.1 \end{bmatrix}$$

Delayed iteration 2



Levy stable distributions 1

★ Theorem: if a pdf f has the form

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi(q) \exp(-iqx) \mathbf{d}q$$

where

$$\log \phi_{\alpha,\beta,\mu,\gamma}(x) = \begin{cases} \mu iq - \gamma |q|^\alpha [1 - i\beta \operatorname{sign}(q) \tan(\pi\alpha/2)] & \alpha \neq 1 \\ \mu iq - \gamma |q| [1 + (2/\pi)i\beta \operatorname{sign}(q) \log |q|] & \alpha = 1 \end{cases}$$

then if $X \sim \phi$ and $Y \sim \phi$ are independent, $X+Y$ has a distribution from the same family

Levy stable distributions 2

- ★ Example: $\alpha = 2, \beta = 0$ gives a Gaussian
- ★ Example: $\alpha = 1, \beta = 0$ gives a Lorentzian
- ★ Problem: if X and Y are Poisson, so is $X+Y$. But for this case

$$\phi(q) = \mu(\exp(iq) - 1)$$

which does not have the required form