Maximum entropy traffic matrix estimation

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CRG presentation 2004 December 03 1600

typeset 2004 December 7 11:23 in pdfLaTeX on a linux system

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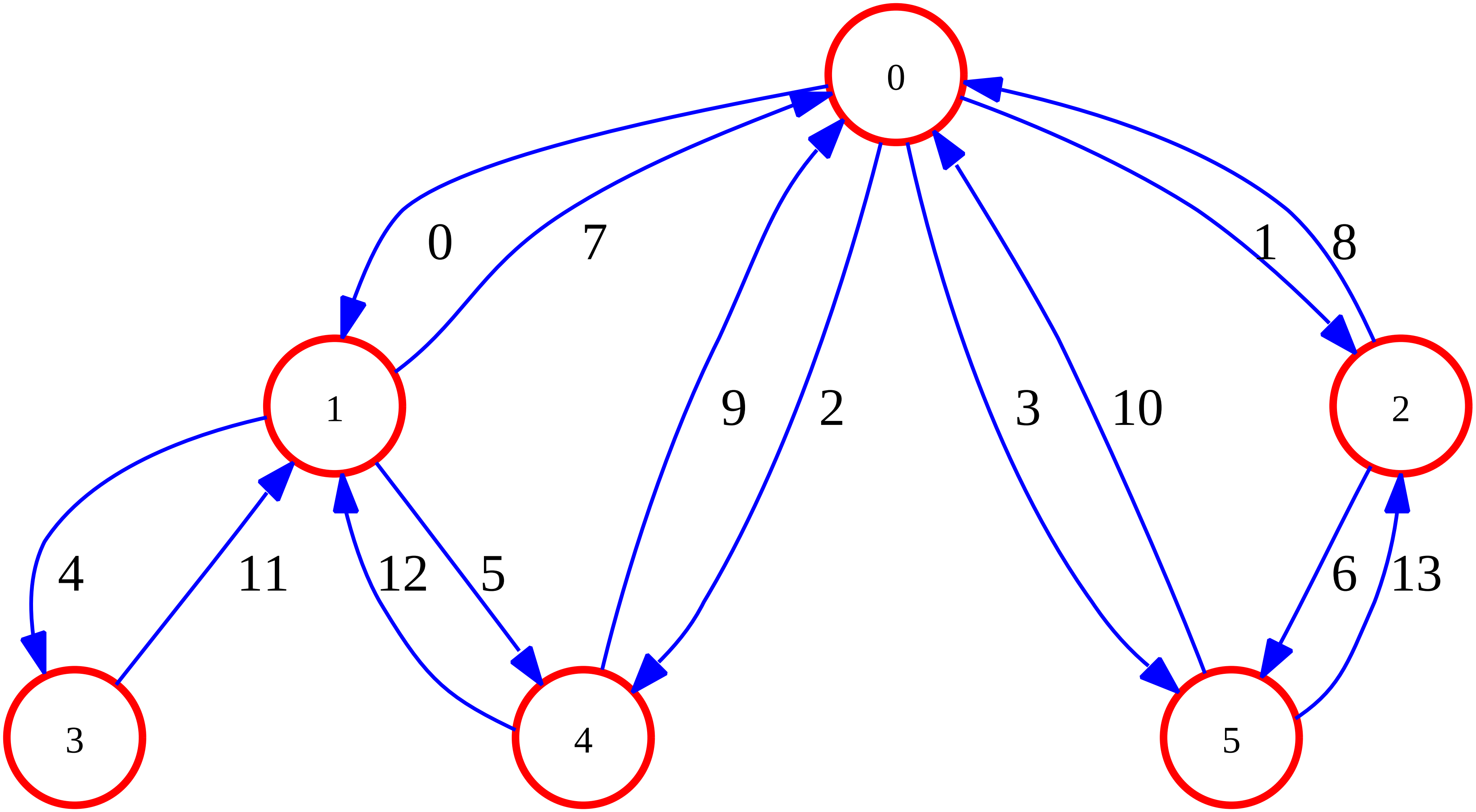
- Note: $x \log(x/g) = (x-g) + (x-g)^2/(2g) + O((x-g)^3)$

Example network



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0.8727	1.1974	0.0000	0.8919	2.1554	5.1278
0.3759	1.0961	0.8919	0.0000	2.1829	0.9470
0.9086	2.9307	2.1554	2.1829	0.0000	2.2887
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- rows are s, columns are d

Example solution

	0	1	2	3	4	5
0	0.000	0.505	0.873	0.376	0.909	0.927
1	0.505	0.000	1.197	1.096	2.931	1.272
2	0.873	1.197	0.000	0.892	2.155	5.128
3	0.376	1.096	0.892	0.000	2.183	0.947
4	0.909	2.931	2.155	2.183	0.000	2.289
5	0.927	1.272	5.128	0.947	2.289	0.000

ns-2 simulation

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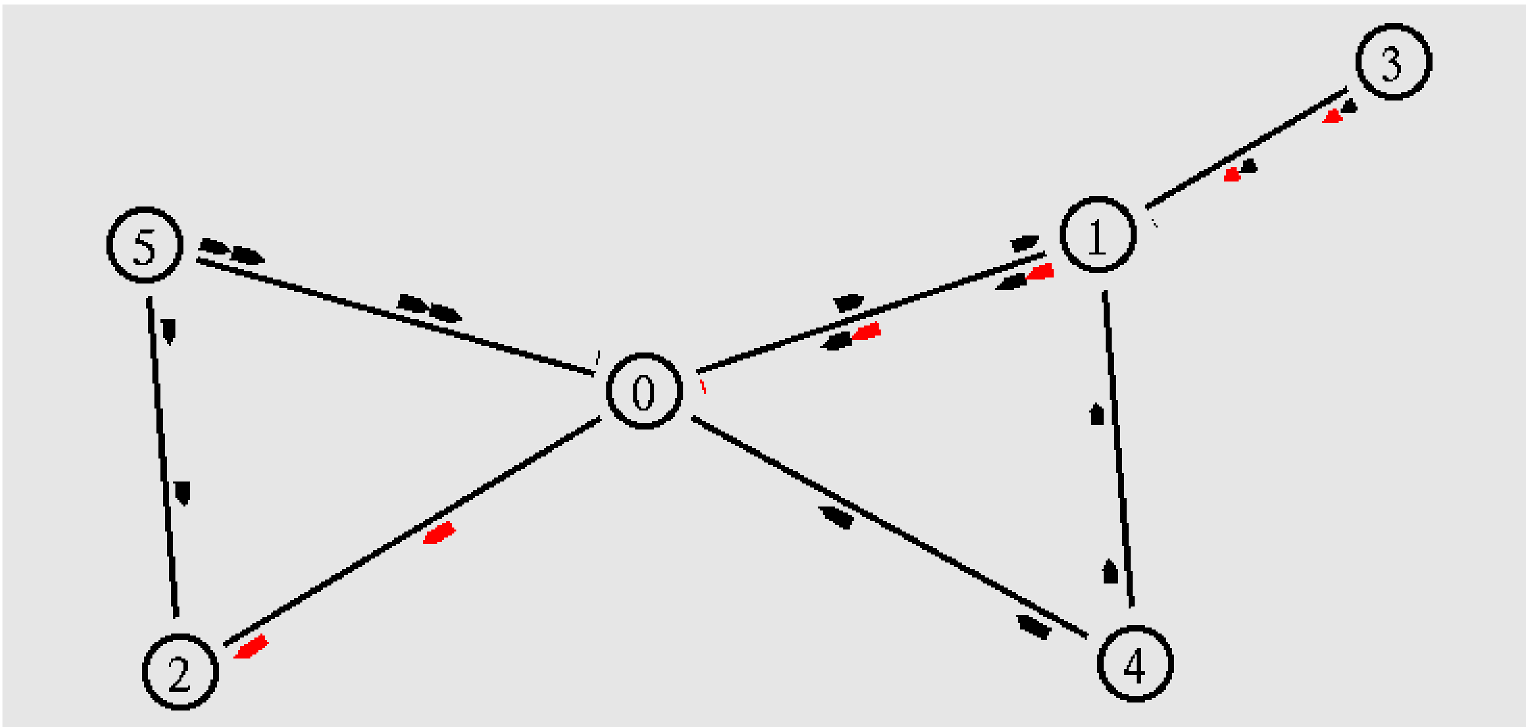
- Can measure all traffic on large network to test accuracy of maxent method



References

[zhang03] Y Zhang, M Roughan, C Lund & D Donoho *An information-theoretic approach to traffix matrix estimation* SIGCOMM03

[ns-2] www.isi.edu/nsnam/ns/